

Beyond mean-field properties of binary dipolar Bose mixtures at low temperatures

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We rigorously analyze the low-temperature properties of homogeneous three-dimensional two-component Bose mixture with dipole-dipole interaction. For such a system the effective hydrodynamic action that governs the behavior of low-energy excitations is derived. The infrared structure of the exact single-particle Green's functions is obtained in terms of macroscopic parameters, namely the inverse compressibility and the superfluid density matrices. Within one-loop approximation we calculate the anisotropic superfluid and condensate densities and give the beyond mean-field stability condition for the binary dipolar Bose gas. A brief variational derivation of the coupled equations that describe macroscopic hydrodynamics of the system in the external non-uniform potential at zero temperature is presented.

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I. INTRODUCTION

During past decade the progress in creation of Bose-Einstein condensates of atoms with large magnetic moments [1–3] opened up a new area in the physics of ultracold gases. The interplay of the anisotropic long-range dipole-dipole interaction and a short-range repulsion leads to the exciting properties of dipolar condensates [4–6].

Interesting, but less studied is the behavior of dipolar Bose mixtures. Depending on the system's parameters the phase diagram of binary dipolar Bose condensates is characterized by the alternation of stable-unstable regions [7] and complicated structures formation [8, 9]. The ground-state properties of low-dimensional systems were also extensively studied [10, 11]. In the quasi-two-dimensional case, where the presence of dipole-dipole interaction leads to the maxon-roton spectrum [12] of elementary excitations, the two-component dipolar Bose system exhibits phase separation caused by the softening of the roton mode [13]. The long-range character of dipolar two-body interaction is responsible for the creation of solitons with a large number of atoms [14]. Even for the binary Bose condensate that contains only one dipolar component a variety of non-trivial phases emerges, including a novel quantized vortices phase [15] in the rotating system and robust supersolid phases [16] in the system on the square lattice.

Recent observation [17] of the droplets formation in dysprosium (¹⁶⁴Dy) condensate will surely stimulate further studies of dipolar Bose systems. These experiments discovered an intriguing phase of the system where the superfluidity is accompanied by the broken translational invariance. Actually, they revealed the role of quantum fluctuations [18, 19] in the formation of new states of

matter.

In the present article by means of hydrodynamic approach we study the properties of two-component dipolar Bose mixtures which, we hope, will soon be implemented. In particular, it is indicated the impact of the beyond mean-field effects on the stability and superfluid properties of two-component dipolar bosonic gases.

II. FORMULATION

The considered model is characterized by the following action

$$S = \int dx \psi_a^*(x) \left\{ \partial_\tau + \frac{\hbar^2}{2m_a} \Delta + \mu_a \right\} \psi_a(x) - \frac{1}{2} \int dx \int dx' \Phi_{ab}(x-x') |\psi_a(x)|^2 |\psi_b(x')|^2, \quad (2.1)$$

here $x = (\tau, \mathbf{r})$ and the summation over repeated indices $a, b = (A, B)$ is assumed. The first term describes two non-interacting sorts of Bose particles with chemical potentials μ_a and the second one takes into account both dipole-dipole interaction as well as short-range repulsion between particles $\Phi_{ab}(x) = \delta(\tau) \Phi_{ab}(\mathbf{r})$

$$\Phi_{ab}(\mathbf{r}) = g_{ab} \delta(\mathbf{r}) + \Phi_{ab}^{(d)}(\mathbf{r}), \quad (2.2)$$

where $\Phi_{ab}^{(d)}(\mathbf{r}) = d_a d_b \frac{1-3z^2/r^2}{r^3}$, i.e. all dipole moments d_A, d_B are assumed to be oriented along z axis. We impose periodic boundary conditions with large volume V on the spatial dependence of complex fields $\psi_a(x)$ and with period $\beta = 1/T$ (T is the temperature) on the imaginary time variable τ .

In order to study the properties of low-energy collective modes we pass to the phase-density representation of ψ -fields [20]

$$\psi_a(x) = \sqrt{n_a(x)} e^{i\phi_a(x)}, \quad \psi_a^*(x) = \sqrt{n_a(x)} e^{-i\phi_a(x)}. \quad (2.3)$$

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For the spatially uniform system we can use the following decomposition of density and phase fields

$$\begin{aligned} n_a(x) &= n_a + \frac{1}{\sqrt{\beta V}} \sum_K e^{iKx} n_K^a, \\ \phi_a(x) &= \frac{1}{\sqrt{\beta V}} \sum_K e^{iKx} \phi_K^a, \end{aligned} \quad (2.4)$$

treating n_a as equilibrium densities of each component [21]. We also define four-momentum $K = (\omega_k, \mathbf{k})$ with condition $\mathbf{k} \neq 0$ imposed on the wave vector, and introduce bosonic Matsubara frequency ω_k .

III. STRUCTURE OF LOW-LYING EXCITATIONS

The hydrodynamic approach allows us to proceed further consideration in the canonical ensemble. The corresponding action after substitution of $n_a(x)$ and $\phi_a(x)$ in Eq. (2.1) reads

$$S = S_0 + S_{int}. \quad (3.5)$$

Here the first term is (δ_{ab} denotes the Kronecker delta)

$$\begin{aligned} S_0 = \text{const} - \frac{1}{2} \sum_K \{ & \omega_k \phi_K^a n_{-K}^a - \omega_k \phi_{-K}^a n_K^a \\ & + \frac{\hbar^2 k^2 n_a}{m_a} |\phi_K^a|^2 + \left[\frac{\hbar^2 k^2}{4m_a n_a} \delta_{ab} + \nu_{ab}(\mathbf{k}) \right] n_K^a n_{-K}^b \}, \end{aligned} \quad (3.6)$$

where $\nu_{ab}(\mathbf{k}) = g_{ab} + 4\pi d_a d_b [k_z^2/k^2 - 1/3]$ is the Fourier transform of two-body potentials; $\text{const} = -\beta V n_a n_b \nu_{ab}(0)/2$ shifts the ground-state energy of the system and $\nu_{ab}(0) = g_{ab}$ should be treated as a direction-averaged value of $\nu_{ab}(\mathbf{k})$ in the $\mathbf{k} \rightarrow \mathbf{0}$ limit. From the latter fact we see that the mean-field thermodynamics of the uniform dipolar Bose system is unaffected by dipole-dipole interaction. It is easy to show by means of simple diagonalization procedure that S_0 is the action of non-interacting Bogoliubov quasiparticles with two branches of excitation spectrum and the last term in Eq. (3.5) takes into account interaction between them

$$\begin{aligned} S_{int} &= \frac{1}{2\sqrt{\beta V}} \sum_{K,Q} \frac{\hbar^2}{m_a} \mathbf{k} \mathbf{q} n_{-K-Q}^a \phi_K^a \phi_Q^a \\ &+ \frac{1}{3!\sqrt{\beta V}} \sum_{K+Q+P=0} \frac{\hbar^2}{8m_a n_a^2} (k^2 + q^2 + p^2) n_K^a n_Q^a n_P^a \\ &- \frac{1}{8\beta V} \sum_{K,Q} \frac{\hbar^2}{2m_a n_a^3} (k^2 + q^2) n_K^a n_{-K}^a n_Q^a n_{-Q}^a. \end{aligned} \quad (3.7)$$

These collisional terms are responsible for the simplest quasiparticle decay processes providing the finiteness of the elementary excitation lifetime.

Previously [22] it was shown how to relate parameters of low-lying excitations with measurable quantities of the

one-component dipolar superfluid. This analysis can be naturally extended on the two-component Bose mixtures with the dipole-dipole interaction. In particular, for all diagrams with two external n_K -lines one obtains

$$D_{nn}^{ab}(K \rightarrow 0) = \partial \mu_a / \partial n_b + 4\pi d_a d_b [k_z^2/k^2 - 1/3]. \quad (3.8)$$

Mentioning that differentiation of every exact vertex function with respect to n_a adds one more zero-momentum n_K^a line to this vertex we conclude

$$D_{nnn}^{abc}(0,0) = \frac{\partial^3 f}{\partial n_a \partial n_b \partial n_c} = \frac{\partial^2 \mu_a}{\partial n_b \partial n_c}, \dots, \quad (3.9)$$

where f is the free energy density of a two-component dipolar Bose system. Note that only second-order vertices exhibit anisotropic behavior in the long-wavelength limit, i.e., depending on the angle between momentum and the direction of the external magnetic field.

Similarly to the one-component case we can derive the second class of identities supposing that each constituent of our system moves with velocity \mathbf{v}_a , which is equivalent to the following gauge transformation $\phi_{A(B)}(x) \rightarrow \phi_{A(B)}(x) - m_{A(B)} \mathbf{r} \mathbf{v}_{A(B)} / \hbar$ of the initial action (3.5). Rotational invariance in the transverse to the external field plane ensures that thermodynamic quantities of the moving Bose system depend only on $\mathbf{v}_a^\perp \mathbf{v}_b^\perp$ and $v_a^z v_b^z$, therefore for the free energy density we have

$$f_{\mathbf{v}} = f + \frac{1}{2} \rho_{ab}^\perp \mathbf{v}_a^\perp \mathbf{v}_b^\perp + \frac{1}{2} \rho_{ab}^z v_a^z v_b^z + \dots, \quad (3.10)$$

where introduced here symmetric matrices ρ_{ab}^z , ρ_{ab}^\perp have the meaning of superfluid mass densities along and in the transverse direction to the dipole orientation, respectively. In the zero-temperature limit the whole liquid is superfluid and consequently the requirement of current conservation leads to the following identities

$$\rho_{AA}^z + \rho_{AB}^z = \rho_{AA}^\perp + \rho_{AB}^\perp = m_A n_A, \quad (A \rightarrow B). \quad (3.11)$$

Taking into account expansion (3.10) and fact that differential operator $\frac{\hbar}{im_{A(B)}} \mathbf{k} \frac{\partial}{\partial \mathbf{v}_{A(B)}}$ acting on the exact vertex function raises the number of $\phi_{A(B)}(K)$ -lines with zero frequency and vanishingly small \mathbf{k} we finally get

$$D_{\phi\phi}^{ab}(K \rightarrow 0) = \frac{\hbar^2 k^2}{m_a m_b} \{ \rho_{ab}^\perp k_\perp^2 / k^2 + \rho_{ab}^z k_z^2 / k^2 \}, \quad (3.12)$$

$$D_{\phi n}^{ab}(K \rightarrow 0) = -D_{n\phi}^{ba}(K \rightarrow 0) = \delta_{ab} \omega_k. \quad (3.13)$$

In principle, absence of infrared divergences in the hydrodynamic description permits to verify the last equality within perturbation theory arguments. Moreover, the behavior of every vertex function is qualitatively reproduced even on the one-loop level. Finally, it should be noted that by using these two differentiation rules one can obtain the long-wavelength asymptotics of arbitrary exact vertex. Particularly, for third-order vertices with two phase and one density lines we find in the limit $K, Q \rightarrow 0$

$$D_{\phi\phi n}^{abc}(K, Q) = -\frac{\hbar^2 \mathbf{k}_\perp \mathbf{q}_\perp}{m_a m_b} \frac{\partial \rho_{ab}^\perp}{\partial n_c} - \frac{\hbar^2 k_z q_z}{m_a m_b} \frac{\partial \rho_{ab}^z}{\partial n_c}, \quad (3.14)$$

which together with (3.9) justify the effective Landau hydrodynamic description of two-component dipolar Bose systems.

Equations (3.8), (3.12) and (3.13) clearly demonstrate the phonon-like behavior of two branches of excitation spectrum in the long-length limit and determine direction-dependent sound velocities in terms of matrices $\chi_{ab}(\mathbf{k}) = D_{nn}^{ab}(K \rightarrow 0)$, $\eta_{ab}(\mathbf{k}) = D_{\phi\phi}^{ab}(K \rightarrow 0)/\hbar^2 k^2$

$$c_{\mathbf{k}}^4 - c_{\mathbf{k}}^2 \text{Sp}\{\eta(\mathbf{k})\chi(\mathbf{k})\} + \det|\eta(\mathbf{k})\chi(\mathbf{k})| = 0. \quad (3.15)$$

Although we discuss the ground-state properties of the system, the above equation is valid for all temperatures up to the superfluidity transition point.

IV. BOSE-EINSTEIN CONDENSATION PHENOMENON

Within our hydrodynamic approach it is easy to find out the infrared structure of the one-particle spectrum. Therefore, the main purpose of this section is to derive exact low-energy asymptotic behavior of the normal

$$G_{ab}(x - x') = -\langle \psi_a(x) \psi_b^*(x') \rangle, \quad (4.16)$$

and anomalous

$$\tilde{G}_{ab}(x - x') = -\langle \psi_a(x) \psi_b(x') \rangle, \quad (4.17)$$

Green's functions which are determined as statistically averaged values of various pairs of ψ -fields. For the spatial dimensionalities higher than two the above functions at equal imaginary time arguments and at large particle spacing tend to the constant

$$G_{ab}(x) = \tilde{G}_{ab}(x) = -\sqrt{n_{0a}n_{0b}}, \quad \tau = 0, r \rightarrow \infty, \quad (4.18)$$

where n_{0a} is the Bose condensate density of sort a . The number of particles in the lower single-particle state is a model-dependent quantity, which in the hydrodynamic approach can be calculated as follows (see [22] for details)

$$\sqrt{n_{0a}} = \lim_{\tau' \rightarrow \tau - 0} \langle \sqrt{n_a(x)} e^{i\phi_a(x')} \rangle|_{\mathbf{r}'=\mathbf{r}}, \quad (4.19)$$

or, equivalently $\lim_{\tau' \rightarrow \tau - 0} \langle e^{-i\phi_a(x)} \sqrt{n_a(x')} \rangle|_{\mathbf{r}'=\mathbf{r}}$. Passing to the four-momentum space

$$\mathcal{G}_{ab}(P) = \int dx e^{-iPx} \{ \sqrt{n_{0a}n_{0b}} + G_{ab}(x) \}, \quad (4.20)$$

$$\tilde{\mathcal{G}}_{ab}(P) = \int dx e^{-iPx} \{ \sqrt{n_{0a}n_{0b}} + \tilde{G}_{ab}(x) \}, \quad (4.21)$$

and taking into account equations (4.16), (4.17), (4.19) and our estimation for the infrared structure of the hydrodynamic action we have

$$\mathcal{G}_{ab}(P \rightarrow 0) = -\sqrt{n_{0a}n_{0b}} \langle \phi_P^a \phi_{-P}^b \rangle, \quad (4.22)$$

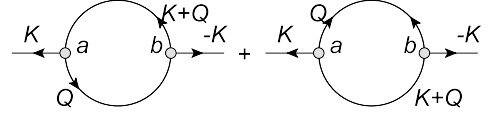


FIG. 1: One-loop corrections to the $D_{\phi\phi}^{ab}(K)$ vertices. Arrows denote phase fields ϕ_K^a and solid lines stand for density fields ρ_K^a .

and $\tilde{\mathcal{G}}_{ab}(P \rightarrow 0) = -\mathcal{G}_{ab}(P \rightarrow 0)$. The applicability of the above equation is not restricted to the low-temperature limit where it generalizes the celebrated Gavoret-Nozières result [23, 24] on the mixture of Bose particles, but it is also valid at finite temperatures in a condensate region. In particular, a zero-frequency limit of this formula is the extension of the Josephson result [25] which due to presence of dipole-dipole interaction possesses intriguing anisotropic behavior.

V. ONE-LOOP CALCULATIONS

The absence of the infrared divergences guarantees that in the weak-coupling limit properties of the system can be described quantitatively even within the first-order perturbation theory. In order to calculate anisotropic superfluid densities of the two-component dipolar mixture we have to collect all diagrams with two external phase lines. On the one-loop level the problem is simple since the correction to every $D_{\phi\phi}^{ab}(K)$ vertex in this approximation is given by two diagrams (see Fig. 1), so for the matrices of superfluid densities we obtain

$$\rho_{ab}^{z(\perp)} = \delta_{ab} m_a n_a - \Delta \rho_{ab}^{z(\perp)}. \quad (5.23)$$

At zero temperature these quantities satisfy the following identities $\Delta \rho_{AA}^{z(\perp)} = \Delta \rho_{BB}^{z(\perp)} = -\Delta \rho_{AB}^{z(\perp)}$ which are consistent with current conservation (3.11). Straightforward calculations yield for $\Delta \rho_{AB}^{z(\perp)}$

$$\Delta \rho_{AB}^z = \frac{2}{V} \sum_{\mathbf{q} \neq 0} \hbar^2 q_z^2 \frac{\varepsilon_A \varepsilon_B}{E_+ E_-} \frac{n_A n_B \nu_{AB}^2}{(E_+ + E_-)^3}, \quad (5.24)$$

$$\Delta \rho_{AB}^\perp = \frac{1}{V} \sum_{\mathbf{q} \neq 0} \hbar^2 (q_x^2 + q_y^2) \frac{\varepsilon_A \varepsilon_B}{E_+ E_-} \frac{n_A n_B \nu_{AB}^2}{(E_+ + E_-)^3}, \quad (5.25)$$

where in order to simplify notation we introduced free-particle dispersion $\varepsilon_a = \hbar^2 q^2 / 2m_a$. We also used notation for the two-body potential $\nu_{ab} = \nu_{ab}(\mathbf{q})$ as well as for two branches of excitation spectrum $E_\pm^2 = (E_A^2 + E_B^2)/2 \pm \sqrt{(E_A^2 - E_B^2)^2/4 + 4\varepsilon_A \varepsilon_B n_A n_B \nu_{AB}^2}$, here $E_A^2 = \varepsilon_A^2 + 2\varepsilon_A n_A \nu_{AA}$ ($E_B = E_{A \rightarrow B}$) is the Bogoliubov spectrum of component A (B).

In the same fashion, by calculating one-loop diagrams contributing to the $D_{nn}^{ab}(K)$ vertices (see Fig. 2) we ob-

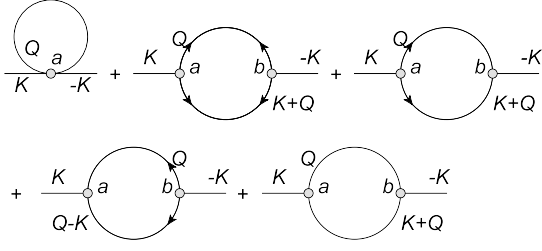


FIG. 2: Diagrams contributing to the $D_{nn}^{ab}(K)$ vertices.

tained corrections to the inverse susceptibilities

$$\frac{\partial \mu_a}{\partial n_b} = g_{ab} + \frac{\partial \Delta \mu_a}{\partial n_b}. \quad (5.26)$$

Omitting details of simple but cumbersome computations we present results obtained in the zero-temperature limit

$$\begin{aligned} \frac{\partial \Delta \mu_A}{\partial n_A} = & \frac{-1}{2V} \sum_{\mathbf{q} \neq 0} \frac{\varepsilon_A^2}{E_+^2 E_-^2} \left\{ \frac{[\nu_{AA} E_B^2 - 2\varepsilon_B n_B \nu_{AB}^2]^2}{E_+ E_- (E_+ + E_-)} \right. \\ & \left. + \frac{[\nu_{AA} (E_+ E_- + E_B^2) - 2\varepsilon_B n_B \nu_{AB}^2]^2}{(E_+ + E_-)^3} \right\} \quad (5.27) \end{aligned}$$

$$\begin{aligned} \frac{\partial \Delta \mu_A}{\partial n_B} = & \frac{-1}{2V} \sum_{\mathbf{q} \neq 0} \frac{\varepsilon_A \varepsilon_B \nu_{AB}^2}{E_+^2 E_-^2} \left\{ \frac{\varepsilon_A^2 \varepsilon_B^2}{E_+ E_- (E_+ + E_-)} \right. \\ & \left. + \frac{(E_+ E_- + \varepsilon_A^2)(E_+ E_- + \varepsilon_B^2)}{(E_+ + E_-)^3} \right\} \quad (5.28) \end{aligned}$$

Of course, one obtains the same result using thermodynamic identities $\frac{\partial \mu_a}{\partial n_b} = \frac{\partial^2}{\partial n_a \partial n_b} \frac{E_0}{V}$, where E_0 is the ground-state energy of two-component dipolar Bose system in the Bogoliubov approximation. It is worth noting that integrals in Eqs. (5.27), (5.28) are ultraviolet divergent. To remove these divergences we use the prescription originally introduced in Ref. [26] for the one-component Bose system with dipolar interaction. The trick is to rewrite the Fourier transform of potentials $\nu_{ab}(\mathbf{k})$ via scattering amplitude at low momenta. Therefore, to the r.h.s. of equations (5.27) and (5.28) we should add $\frac{m}{\hbar^2} \int \frac{d\mathbf{q}}{(2\pi)^3} \nu_{AA}^2 / q^2$ and $\frac{2m_A m_B}{\hbar^2 (m_A + m_B)} \int \frac{d\mathbf{q}}{(2\pi)^3} \nu_{AB}^2 / q^2$, respectively. The same result is obtained with the help of usual dimensional regularization procedure.

In the dilute limit the leading-order contribution to the condensate depletion can be calculated in the closed form by making use of Taylor series expansion of exponential and square root factors in Eq. (4.19)

$$\begin{aligned} \frac{n_{0A}}{n_A} = & 1 - \langle \phi_A^2(x) \rangle - \frac{1}{4n_A^2} \langle n_A^2(x) \rangle \\ & + \frac{i}{n_A} \lim_{\tau' \rightarrow \tau-0} \langle \phi_A(x') n_A(x) \rangle|_{\mathbf{r}'=\mathbf{r}}, \quad (5.29) \end{aligned}$$

and similarly for the sort B . Performing integration over Matsubara frequency for $\Delta n_A = n_A - n_{0A}$ we obtain

$$\Delta n_A = \frac{1}{2V} \sum_{\mathbf{q} \neq 0} \left\{ \frac{\varepsilon_A + n_A \nu_{AA}}{E_+ + E_-} \left[1 + \frac{E_B^2}{E_+ E_-} \right] - 1 \right\} \quad (5.30)$$

The above result in the limit of vanishingly small inter-species interaction coincides with that of Ref. [27].

For specific calculations we will restrict ourselves by considering only mixtures with equal mass of different sorts of particles. This is quite a reasonable approximation for experimentally relevant two-component ^{162}Dy - ^{164}Dy and ^{164}Dy - ^{162}Er systems. Particularly, in this equal-mass limit $m_A = m_B = m$ two branches of the excitation spectrum take a very simple form $E_{\pm}^2 = \hbar^4 q^4 / 4m^2 + \hbar^2 c_{\pm}^2 q^2$ providing that all thermodynamic parameters of a binary dipolar Bose mixture at zero temperature can be written in terms of one-dimensional integrals (see Appendix for details). Recent experimental predictions [28, 29] for the scattering lengths of dysprosium and erbium condensates clearly show that the above mentioned homogeneous mixtures are unstable towards collapse. Therefore, in order to demonstrate the impact of the dipole-dipole interaction on the equilibrium properties of two-component Bose gases we adopt simplified model with $\det g_{ab} = 0$ and $4\pi d_A^2 / 3g_{AA} = 4\pi d_B^2 / 3g_{BB} = \varepsilon < 1$. This model is of particular interest because such a system on the mean-field level is at the threshold of phase separation and only the presence of quantum effects recovers the thermodynamic stability of mixed state. Introducing s -wave scattering lengths $l_A = g_{AA} m / 4\pi \hbar^2$ ($l_B = l_{A \rightarrow B}$) and typical notation $\mathcal{Q}_s(\varepsilon) = \int_0^1 dx [1 + \varepsilon(3x^2 - 1)]^{s/2}$ we are in position to write down condensate depletion

$$\Delta n_A = \frac{8}{3\sqrt{\pi}} n_A l_A \sqrt{n_A l_A + n_B l_B} \mathcal{Q}_3(\varepsilon), \quad (5.31)$$

superfluid densities

$$\begin{aligned} \Delta \rho_{AB}^z / m = & \frac{64}{45\sqrt{\pi}} \frac{n_A l_A n_B l_B}{\sqrt{n_A l_A + n_B l_B}} \\ & \times \left\{ \frac{1}{\varepsilon} \mathcal{Q}_5(\varepsilon) - \frac{1-\varepsilon}{\varepsilon} \mathcal{Q}_3(\varepsilon) \right\}, \quad (5.32) \end{aligned}$$

$$\begin{aligned} \Delta \rho_{AB}^{\perp} / m = & \frac{32}{45\sqrt{\pi}} \frac{n_A l_A n_B l_B}{\sqrt{n_A l_A + n_B l_B}} \\ & \times \left\{ \frac{1+2\varepsilon}{\varepsilon} \mathcal{Q}_3(\varepsilon) - \frac{1}{\varepsilon} \mathcal{Q}_5(\varepsilon) \right\}, \quad (5.33) \end{aligned}$$

and corrections to the inverse compressibility matrix

$$\frac{1}{g_{AA}} \frac{\partial \Delta \mu_A}{\partial n_A} = \frac{16}{\sqrt{\pi}} l_A \sqrt{n_A l_A + n_B l_B} \mathcal{Q}_5(\varepsilon), \quad (5.34)$$

$$\frac{1}{g_{AB}} \frac{\partial \Delta \mu_A}{\partial n_B} = \frac{16}{\sqrt{\pi}} \sqrt{l_A l_B} \sqrt{n_A l_A + n_B l_B} \mathcal{Q}_5(\varepsilon). \quad (5.35)$$

As it is seen from the above calculations the mixture is always stable for $l_A \neq l_B$. The sound velocity $c_-^2 \simeq n_A n_B \det \chi(\mathbf{k}) / (mc_+)^2$ of the lower branch of excitation spectrum which is fully determined by the one-loop result (5.34), (5.35) reveals isotropic behavior and increases with increasing of the strength of dipole-dipole interaction.

VI. SUPERFLUID HYDRODYNAMICS

In the following we consider only the low-temperature limit completely ignoring densities of normal component in the superfluid. To obtain the equations of macroscopic hydrodynamics we use Zisel prescription adopted from [30]. The underlying idea is the formulation of the local non-uniform densities $n_a(\mathbf{r}, t)$ and velocities fields $\mathbf{v}_a^\perp(\mathbf{r}, t)$, $v_a^z(\mathbf{r}, t)$ for two-component dipolar bosons. The appropriate Lagrangian density $\mathcal{L} = \mathcal{K} - \mathcal{U}$ necessarily contains kinetic energy term (see Eq. (3.10)) $\mathcal{K} = \frac{1}{2}\rho_{ab}^\perp[n]\mathbf{v}_a^\perp(\mathbf{r}, t)\mathbf{v}_b^\perp(\mathbf{r}, t) + \frac{1}{2}\rho_{ab}^z[n]v_a^z(\mathbf{r}, t)v_b^z(\mathbf{r}, t) + \dots$, where matrices $\rho_{ab}^\perp[n]$, $\rho_{ab}^z[n]$ (through the local densities of each sort of particles) depend on spatial coordinates and time. The second term in \mathcal{L} is the internal energy per volume of the two-component Bose system with dipole-dipole interaction in the external potentials $U_a(\mathbf{r})$

$$\mathcal{U} = \epsilon[n] + U_a(\mathbf{r})n_a(\mathbf{r}, t) + \frac{1}{2} \int d\mathbf{r}' \Phi_{ab}^{(d)}(\mathbf{r}' - \mathbf{r}) n_a(\mathbf{r}, t) n_b(\mathbf{r}', t), \quad (6.36)$$

where $\epsilon[n]$ is energy density of the uniform system after substitution $n_a \rightarrow n_a(\mathbf{r}, t)$. When minimizing action $\mathcal{A} = \int dt \int d\mathbf{r} \mathcal{L}$ one should take into account the local particle number conservation laws

$$\partial_t n_a + \frac{1}{m_a} \text{div} \mathbf{j}_a = 0, \quad (6.37)$$

(from now on we do not write the dependence on \mathbf{r}, t explicitly) with currents defined by $\mathbf{j}_a = (\rho_{ab}^\perp[n]\mathbf{v}_a^\perp, \rho_{ab}^z[n]v_b^z)$. Performing these simple calculations we obtain equations that determine conditional extremum of \mathcal{A}

$$\partial_t \mathbf{v}_a = -\frac{1}{m_a} \nabla \left\{ \mu_a[n] + \int d\mathbf{r}' \Phi_{ab}^{(d)}(\mathbf{r} - \mathbf{r}') n'_b + U_a(\mathbf{r}) + \frac{1}{2} \frac{\partial \rho_{bc}^\perp[n]}{\partial n_a} \mathbf{v}_b^\perp \mathbf{v}_c^\perp + \frac{1}{2} \frac{\partial \rho_{bc}^z[n]}{\partial n_a} v_b^z v_c^z \right\}, \quad (6.38)$$

which have to be solved together with (6.37). Few comments are in order to outline the limits of applicability of the above hydrodynamic equations. First of all, the total kinetic energy \mathcal{K} of two-component superfluid despite one-component case is not a quadratic form over velocities even at very low temperatures. Therefore omitting higher-order terms we restrict our consideration to the distant hydrodynamic region close to the equilibrium. In practice, using procedure of section 5 it is not hard to obtain these quadruple, etc. terms by calculating appropriate vertices $D_{\phi\phi\phi\phi}^{abcd}(K, Q, P)$, \dots , but in the experimentally relevant dilute limit their contribution are negligibly small. Secondly, this semi-phenomenological formulation of the macroscopic hydrodynamics is quasi-classical by its nature which suggests the external potential $U_a(\mathbf{r})$ to be a smoothly varying function of coordinates, since only in this case the so-called quantum

pressure term $\hbar^2(\nabla n_a)^2/(m_a n_a^2)$ is smaller than chemical potential $\mu_a[n]$. Moreover, in the absence of superfluid flow ($\mathbf{v}_a = 0$) from (6.38) we get the Thomas-Fermi stability condition for the two-component system in the non-uniform external potential. However, the obtained system of coupled equations can be easily used to detect the formation of droplets with large number of particles [18] in mixtures with dipole-dipole interaction. It is worth noting that the linearized equations (6.37), (6.38) correctly describe the propagation of sound-waves in the homogeneous ($U_a(\mathbf{r}) = 0$) system with velocities given by Eq. (3.15).

VII. CONCLUSIONS

To summarize, we have studied the properties of binary dipolar Bose condensates at zero temperature. Making use of hydrodynamic description in terms of density and phase fluctuations we have found the connection between infrared anisotropic behavior of one-particle Green's functions and dynamic structure factors with macroscopic parameters of two-component superfluids. Within this approach the matrices of superfluid densities and inverse susceptibilities are calculated in the one-loop approximation for a model of dipolar Bose gas with the short-range repulsion.

Additionally we pointed out on the correct way to calculate the condensate density of interacting bosons in the Popov's hydrodynamic formulation. The impact of the dipole-dipole interaction on the condensate depletion in the two-component Bose gas is examined.

Finally, using variational approach we have considered the problem of macroscopic motion of two-component dipolar superfluids at very low temperatures. The obtained system of coupled hydrodynamic equations can be used to describe future experiments with binary dipolar Bose condensates.

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VIII. APPENDIX

In this section we present some technical details of our first order perturbative calculations. In the case when masses of two species of particles are equal to each other the inverse susceptibilities of the binary dipolar Bose gas can be written via integrals over polar angle

$$\frac{\partial \Delta \mu_A}{\partial n_B} = \frac{1}{3\pi^2} \int_0^\pi d\theta \sin \theta \frac{mc_+^2}{n_A n_B} \left(\frac{mc_+}{\hbar} \right)^3 \times \gamma_{AB}^4 f_1(\gamma), \quad (8.39)$$

$$\frac{\partial \Delta \mu_A}{\partial n_A} = \frac{1}{3\pi^2} \int_0^\pi d\theta \sin \theta \frac{mc_+^2}{n_A^2} \left(\frac{mc_+}{\hbar} \right)^3 \times \{ \gamma_A^4 f_1(\gamma) - 2\gamma_A^2 \gamma^2 f_2(\gamma) - \gamma^4 f_3(\gamma) \}, \quad (8.40)$$

where in order to shorten notations we introduced following functions

$$f_1(\gamma) = \frac{3\gamma^4 + 9\gamma^3 + 11\gamma^2 + 9\gamma + 3}{(1 + \gamma)^3},$$

$$f_2(\gamma) = \frac{\gamma^2 + 3\gamma + 1}{(1 + \gamma)^3}, \quad f_3(\gamma) = \frac{1}{(1 + \gamma)^3}.$$

We also use abbreviation for $\gamma = c_-/c_+$, $\gamma_A = c_A/c_+$ and $\gamma_{AB}^4 = n_A n_B \nu_{AB}^2(\mathbf{k})/m^2 c_+^4$. Here $c_\pm^2 = (c_A^2 + c_B^2)/2 \pm \sqrt{(c_A^2 - c_B^2)^2/4 + n_A n_B \nu_{AB}^2(\mathbf{k})/m^2}$ are sound velocities of two branches of the excitation spectrum of the system and $c_A^2 = n_A \nu_{AA}(\mathbf{k})/m$ ($c_B = c_{A \rightarrow B}$) is the mean-field sound velocity of each sort along. In the same fashion

the matrix elements of superfluid densities read

$$\Delta \rho_{AB}^z/m = \frac{4}{15\pi^2} \int_0^\pi d\theta \sin \theta \cos^2 \theta \left(\frac{mc_+}{\hbar} \right)^3 \times \gamma_{AB}^4 f_2(\gamma), \quad (8.41)$$

$$\Delta \rho_{AB}^\perp/m = \frac{2}{15\pi^2} \int_0^\pi d\theta \sin \theta \sin^2 \theta \left(\frac{mc_+}{\hbar} \right)^3 \times \gamma_{AB}^4 f_2(\gamma). \quad (8.42)$$

Finally, for the condensate depletion of sort B we find

$$\Delta n_B = \frac{1}{6\pi^2} \int_0^\pi d\theta \sin \theta \left(\frac{mc_+}{\hbar} \right)^3 \times \left\{ \frac{1 - \gamma^5}{1 - \gamma^2} - \gamma_A^2 \frac{1 - \gamma^3}{1 - \gamma^2} \right\}. \quad (8.43)$$

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